SEE MODEL QUESTION PAPER-1 UG

First Semester B.E. Degree Examination, April - 2021

Calculus and Linear Algebra

Time: 3 hrs. **Course Code: 20MAT11** **Max. Marks: 100**

Note: Answer ONE full question from each module.

		MODULE - 1	Marks
1	a.	Find nth derivative of $y=\frac{x^2}{2x^2+7x+6}$	6
	b.	Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$	7
	c.	Find the pedal equation of the curve $r^m \cos m\theta = a^m$	7
		OR	
2	a.	Using Maclaurin's series, expand $\sqrt{(1+\sin 2x)}$ up to the terms containing x^4	6
	b.	Find the angle of intersection of the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$	7
	c.	Show that the radius of curvature at $(a, 0)$ on the curve $y^2 = a^2(a-x)/x$ is $a/2$	7
		MODULE – 2	
3	a.	If $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$	6
	b.	If $u = x^2 - y^2, v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$	7
	c.	Expand $x^2y + 3y - 2$ in powers of (x-1) and (y+2) using Taylor's theorem up to third degree terms	7
		OR	
4	a.	If $z = e^{ax+by}f(ax-by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$	6
	b.	If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$	7
	c.	Find maximum and minimum values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$	7
		MODULE - 3	
5	a.	Evaluate by changing the order of integration $\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$	6
	b.	Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$	7
	c.	Prove that with usual notations $\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$	7
		OR	
6	a.	Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dy\,dx$ by changing to polar co-ordinates	6

	b.	Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$	7
	c.	Show that $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} = \pi$	7

MODULE - 4

7	a.	Find directional derivative of $\emptyset = x^2yz + 4xz^2$ at the point (1, -2,1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$	6
	b.	Evaluate div $\vec{F}$ and curl $\vec{F}$ at the point (1,2,3) given $\vec{F} = grad(x^3y + y^3z + z^3x - x^2y^2z^2)$	7
	c.	Show that $div(grad\, r^n) = n(n+1)\, r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$	7

OR

8	a.	Using Green's theorem, evaluate $\int_c (3x-8y^2)dx + (4y-6xy)dy$ where C is the boundary of the region bounded by $x = 0, y = 0$ & $x + y = 1$	6
	b.	Employ Gauss divergence theorem to evaluate $\iint_s \vec{F}.\hat{n}ds$ where $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$	7
	c.	Use Stoke's theorem to evaluate $\int_s curl\, \vec{F} \cdot \hat{n}ds$, where $\vec{F} = y\hat{\imath} + (x - 2zx)\hat{\jmath} - xy\hat{k}$ S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane	7

MODULE - 5

9	a.	Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by elementary row transformations	6
	b.	Reduce the matrix to diagonal form given $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$	7
	c.	Solve the system of equations 20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25 by Gauss-Seidel method.	7

OR

10	a.	Find Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	6
	b.	Solve the system of equations x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40 by Gauss Elimination method.	7
	c.	Employ LU decomposition method to solve the system of equations 3x + 2y + 7z = 4, 2x + 3y + z = 5, 3x + 4y + z = 7.	7

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