SEE MODEL QUESTION PAPER

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21MAT11

First Semester B.E. Degree Examination, March- 2022

Linear Algebra and Calculus

Time: 3 hrs. **Max. Marks: 100**

Note: Answer any Five full questions, choosing ONE full question from each module.

Q. No.		MODULE - 1	Marks
1	a	Find the rank of the matrix by applying elementary row transformations $A=\begin{pmatrix}1 & 2 & -2 & 3\\ 2 & 5 & -4 & 6\\ -1 & -3 & 2 & -2\\ 2 & 4 & -1 & 6\end{pmatrix}$	(7 marks)
	b	Investigate for what values of λ and μ the simultaneous equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i) no solution, (ii) a unique solution, (iii) an infinite number of solutions.	(7 marks)
	c	Solve by Gauss Jordan method $\begin{aligned}4x_1-2x_2+6x_3&=8\\ x_1+x_2-3x_3&=-1\\ 15x_1-3x_2+9x_3&=21\end{aligned}$	(6 marks)
2	a	Find the Eigen values and the corresponding Eigen vectors of the matrix $A=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$	(7 marks)
	b	Solve by Gauss elimination $\begin{aligned}2x+y+z&=10\\ 3x+2y+3z&=18\\ x+4y+9z&=16\end{aligned}$	(7 marks)
	c	Solve by Gauss- Seidel method: $\begin{aligned}x+y+52z&=110\\ 27x+6y-z&=85\\ 6x+15y+2z&=72\end{aligned}$, Carryout four iterations	(6 marks)
		MODULE - 2	
3	a	Obtain the nth derivative of $e^{ax}\cos(bx+c)$ and hence find nth derivative of $e^{2x}\cos(3x+5)$	(7 marks)

	b	Verify Rolle's theorem for the function $e^x(\sin x - \cos x)$ in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$	(7 marks)
	c	If x is positive, show that $x > \log(1+x) > x - \frac{1}{2}x^2$.	(6 marks)
4	a	Obtain the nth derivative of $\log(ax+b)$ and hence find nth derivative of $\log(4x^2-1)$	(7 marks)
	b	If $f(x) = \sin^{-1} x, 0 < a < b < 1$, use Mean value theorem to prove that $\frac{b-a}{\sqrt{(1-a^2)}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{(1-b^2)}}$	(7 marks)
	c	Using Maclaurin's expansion series prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - - -$	(6 marks)
		MODULE - 3	
5	a	Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$	(7 marks)
	b	Find the angle between the curves r = a(1+sinθ) and r = a(1-sinθ).	(7 marks)
	c	Find the pedal equation of $\frac{2a}{r} = 1 - \cos\theta$.	(6 marks)
6	a	Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$	(7 marks)
	b	Find the angle between the radius vector and the tangent for the curve r = a(1 – cos θ). Also find the slope of the curve at θ = π/6.	(7 marks)
	c	Find the radius of curvature of the curve $x^3+y^3 = 2a^3$ at the point (a, a)	(6 marks)
		MODULE - 4	
7	a	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ than prove that i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$	(7 marks)
	b	If $x + y + z = u, \; y + z = uv$ and $z = uvw$ then find the value of $\frac{\partial(x, y, z)}{\partial(u, v, w)}$	(7 marks)
	c	Find the extreme value of f(x, y) = $x^3 y^2(1 - x - y)$.	(6 marks)
8	a	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ find the value of $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z}$	(7 marks)
	b	If $x = r\cos\theta, \; y = r\sin\theta$, evaluate $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$	(7 marks)
	c	Find the maximum and minimum values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$	(6 marks)

		MODULE - 5	
9	a	If $\vec{F} = grad\left(x^3y + y^3z + z^3x - x^2y^2z^2\right)$ then find div $\vec{F}$ and curl $\vec{F}$ at (1, 2, 3)	(7 marks)
	b	If $\vec{F} = (x+y+az)i + (bx+2y-z)j + (x+cy+2z)k$, find a, b, c such that $curl\vec{F} = \vec{0}$ and find ϕ such that $\nabla\phi = F$.	(7 marks)
	c	If $\vec{F} = (x+y+1)i + j - (x+y)k$ then prove that $\vec{F}\cdot curl\vec{F} = 0$.	(6 marks)
10	a	Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point (2, -1, 2)	(7 marks)
	b	Find a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, -1) has maximum magnitude of 64 in the direction of z-axis.	(7 marks)
	c	If $\vec{\upsilon} = 3xy^2z^2i + y^3z^2j - 2y^2z^3k$ and $\vec{F} = \left(x^2 - yz\right)i + \left(y^2 - zx\right)j + \left(z^2 - xy\right)k$ the prove that $\vec{\upsilon}$ is solenoidal and $\vec{F}$ is irrotational	(6 marks)

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