SEE MODEL QUESTION PAPER-2 UG

First Semester B.E. Degree Examination, April - 2021

Calculus and Linear Algebra

Time: 3 hrs. **Course Code:20MAT11** **Max. Marks: 100**

Note: Answer ONE full question from each module.

		MODULE - 1	Marks
1	a.	Find nth derivative of $\sin^3 x \cos^2 x$	6
	b.	Find the angle of intersection of the curves $r = \dfrac{a}{(1+\cos\theta)}$ and $r = \dfrac{b}{(1-\cos\theta)}$	7
	c.	For the curve $y = \dfrac{ax}{a+x}$, show that $\left(\dfrac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\dfrac{x}{y}\right)^2 + \left(\dfrac{y}{x}\right)^2$.	7
		OR	
2	a.	Using Maclaurin's series, expand $\log(1+\cos x)$ up to the terms containing x^4	6
	b.	Evaluate (i) $\underset{x\to 0}{Lt}\ (\cot x)^{\frac{1}{\log x}}$ (ii) $\underset{x\to\frac{\pi}{2}}{Lt}\ (2x\tan x - \pi\sec x)$	7
	c.	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$	7
		MODULE – 2	
3	a.	Find $\dfrac{du}{dt}$ at $t=0$ if $u = e^{x^2+y^2+z^2}$ *and* $x = t^2+1$, $y = t\cos t$, $z = \sin t$	6
	b.	If $u = x+y+z,\ uv = y+z,\ uvw = z$, show that $\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.	7
	c.	Find extreme values of $f(x,y) = x^4 + y^4 - 2(x-y)^2$	7
4	a.	If $u = e^{x^3+y^3}$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3u\ \log u$	6
	b.	If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$.	7
	c.	Expand $\sin x.\cos y$ about the point (0,0) using Taylor's theorem up to the term containing 3rd degree.	7
		MODULE - 3	
5	a.	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dy\,dx$ by changing to polar co-ordinates.	6
	b.	Find the volume of the tetrahedron by the plane $x=0,\ y=0,\ z=0\ \&\ \dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ by using triple integration.	7

c. Prove that $\int_0^1 \frac{x^2\,dx}{\sqrt{(1-x^4)}} \times \int_0^1 \frac{dx}{\sqrt{(1+x^4)}} = \frac{\pi}{4\sqrt{2}}$. 7

OR

6 a. Evaluate by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\,dx.$ 6

b. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz.dy.dx$ 7

c. Evaluate $\int_0^1 x^{\frac{3}{2}} (1-x)^{\frac{1}{2}} dx.$ by expressing in terms of beta and gamma functions. 7

MODULE – 4

7 a. Find directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at the point (-1,1,2) has maximum magnitude of 32 units in the direction parallel to y-axis find a, b, c. 6

b. Show that $\vec{F} = \frac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. 7

c. If $F = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ find a, b, c such that $Curl\ \vec{F} = 0$ and find ϕ such that $\nabla\phi = F.$ 7

OR

8 a. Using Green's theorem evaluate $\int_C (xy + y^2)dx + x^2\,dy$ where C is the boundary of the region bounded by $y = x$ and $y = x^2$. 6

b. Employ Gauss divergence theorem to evaluate $\int_s \vec{A}.\hat{n}.ds$ where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and 's' is the surface area of the sphere $x^2 + y^2 + z^2 = a^2$. 7

c. Use Stoke's theorem to evaluate $\int_s curl\ \vec{F}.\hat{n}\,ds$ for $\vec{F} = (y - z - 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ where S is the surface of the cube formed by the planes $x = 0,\ y = 0,\ x = 2, y = 2\ and\ z = 2$ 7

MODULE - 5

9 a. Find the rank of $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by elementary row transformations. 6

b. Reduce the matrix to diagonal form given $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ and hence find A^4. 7

c. Solve the system of equations 2x + y + 4z = 12, 4x+11y-z = 33, 8x-3y + 2z = 20 by Gauss Elimination method. 7

10 a. Find Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 6

b. Solve the system of equations x +4y +2z =15 , x + 2y+5z = 20, 5x+2y + z = 12 by Gauss-Seidel method 7

c. Solve the system of equations, x + 5y +z = 14, 2x +y + 3z = 13, 3x + y+ 4z = 17 by LU decomposition method 7

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